RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2018

FIRST YEAR (BATCH 2017-20)

Date : 24/05/2018 **MATH FOR IND. CHEMISTRY (General)**

Time : 11.00 am – 2.00 pm Paper : II Full Marks : 75

[Use one Answer Book for <u>Group A,B & C</u> and another Answer Book for <u>Group-D</u>]

<u>Group-A</u>

Answer <u>any three</u> questions [3×5]

1. Prove that the necessary and sufficient condition for three distinct points with position vectors $\vec{a},\vec{b}$ and $\vec{c}$ to be collinear is that there exist three scalars *x*, *y* and *z*, not all zero, such that

   $x\vec{a}+y\vec{b}+z\vec{c}=0$, where $x+y+z=0$. 5

2. a) Show that $\vec{\alpha}\cdot\vec{\beta}+\vec{\beta}\cdot\vec{\gamma}+\vec{\alpha}\cdot\vec{\gamma}=-45$, where $\vec{\alpha},\vec{\beta},\vec{\gamma}$ are vectors such that $\vec{\alpha}+\vec{\beta}+\vec{\gamma}=\vec{0}$ and $|\vec{\alpha}|=7, |\vec{\beta}|=4, |\vec{\gamma}|=5$. 3

   b) Find the vector equation of the straight line passing through the points $A\ 1,1,3$, $B\ 2,1,4$. 2

3. Prove that $\vec{a}+\vec{b},\vec{b}+\vec{c},\vec{c}+\vec{a}$ are coplanar iff $\vec{a}, \vec{b},\vec{c}$ are coplanar. 5

4. Find the vector equation of the plane passing through three non-collinear points whose position vectors are $\vec{a}$, $\vec{b}$ and $\vec{c}$. 5

5. A force of 15 units acts in the direction of the vector $\vec{i}-2\vec{j}+2\vec{k}$ and passes through the point $2\vec{i}-2\vec{j}+2\vec{k}$. Find the moment of the force about the point $\vec{i}+\vec{j}+\vec{k}$. 5

<u>Group-B</u>

Answer <u>any five</u> questions [5×5]

6. a) State Cauchy's General principal of convergence (for series). 2

   b) If $\widehat{x}_n=\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+\cdots+\frac{1}{n(n+1)}$, then show that x_n is a bounded monotonic increasing sequence. 3

7. a) Examine the convergence of the series $1+\frac{1}{1!}+\frac{1}{2!}+\frac{1}{3!}+\cdots+\frac{1}{(n-1)!}+\cdots$. 3

   b) State D'Alembert's ratio test for convergence or divergence of a series of positive terms. 2

8. Test for convergence of the series $\sum\frac{n^2-1}{n^2+1}x^n,\ x>0$. 5

9. If a function f is (i) continuous on [*a*, *b*], (ii) derivable on (*a*, *b*) and (iii) first order derivative with respect to x i.e. $f'(x)=0$ for all $x\in\ a,b$, then prove that $f(x)$ is constant on a,b . 5

10. If a function f is such that its first order derivative f' is continuous on a,b and derivable on a,b , then show that there exists a number c between a and b such that $f(b)=f(a)+(b-a)\ f'(a)+\frac{1}{2}(b-a)^2\ f''(c)$.

    where $f''(c)$ is the second order derivative at the point *c*. 5

11. Show that the maximum value of $x+\dfrac{1}{x}$ is less than its minimum value. 5

12. Evaluate: $\lim\limits_{x\to 1-} 1-x^2 \; {}^{\frac{1}{\log(1-x)}}$. 5

13. Using Lagrange's undetermined multiplier, find the maximum value of x^3y^2 subject to the constraint $x+y=1$. 5

Group-C

Answer <u>any two</u> questions [2×5]

14. Obtain the reduction formula for $\int \tan^n x\, dx$. Using this formula, obtain the value of $\int_0^{\frac{\pi}{4}} \tan^n x\, dx$, where, n is a positive integer. 3+2

15. From the definition of definite integrals, evaluate $\underset{n\to\infty}{Lt}\left\{\dfrac{1}{n+1}+\dfrac{1}{n+2}+\cdots+\dfrac{1}{n+n}\right\}$. 5

16. Show that $\displaystyle\int \frac{dx}{f(x)} = \frac{1}{n!}\left[\log|x| + \sum_{r=1}^{n} -1^{\;r}\; {}^nC_r \log|x+r|\right] + k$ where, $f(x) = x(x+1)(x+2)\cdots(x+n)$ and k is constant of integration. 5

Group-D

Answer <u>any five</u> questions [5×5]

17. a) What is the chance that a leap year selected at random will contain 53 Wednesday? 2

    b) Define Random experiment and Mutually exclusive events. 3

18. If two events A and B are independent, show that A and B^c are independent and hence that A^c and B^c are independent. 5

19. a) Find the value of K (constant) such that

    $f(x) = Kx\; 1-x \quad$ for $0<x<1$

    $\quad = 0 \quad$ elsewhere

    is a probability density function. 2½

    b) A random variable X can assume the values $-1, 0, 1$ with probabilities $\dfrac{1}{3}, \dfrac{1}{2}$ and $\dfrac{1}{6}$ respectively. Determine the distribution function. 2½

20. a) A special un-biased dice with $n+1$ faces is rolled. It's faces are marked by the number $0, \dfrac{1}{n}, \dfrac{2}{n}, \cdots, \dfrac{n-1}{n}, \dfrac{n}{n}$. If X denotes the number shown, then find the expectation of X and also find the expectation of $\left(X-\dfrac{1}{2}\right)^2$. 4

    b) What is probability distribution? 1

21. a) State Poisson distribution. 2

    b) A radio active source emits on the average 2.5 particles per seconds. Calculate the probability that 2 or more particles will be emitted in an interval of 4 seconds. 3

22. If X has Binomial distribution with parameter n (no. of trials) and p (probability of success) then show (i) it's mean is np and (ii) it's variance is npq, where $q = 1 - p$. 5

23. a) If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and standard deviation Rs. 70 and Rs. 5 respectively. Find the expected number of workers whose weekly wages are between Rs. 66 and Rs. 72.

[Given that $\frac{1}{\sqrt{2\pi}}\int_0^z e^{\frac{-t^2}{2}}\,dt = 0.1554$ and 0.2881 according as $z = 0.4$ and $z = 0.8$ respectively.] 4

b) Explain the statement "Poisson distribution be considered as an approximation of the Binomial distribution". 1

24. a) In a precision bombing attack there is a 50% chance that any one bomb will strike the target. Two direct hits are required to destroy the target completely. How many bombs must be dropped to give a 99% chance or better of completely destroying the target? 2½

b) A random variable X has the density function $f(x) = \frac{a}{x^2+1}, -\infty < x < \infty$

Find the value of a and find the probability that X^2 lies between $\frac{1}{3}$ and 1. 2½

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